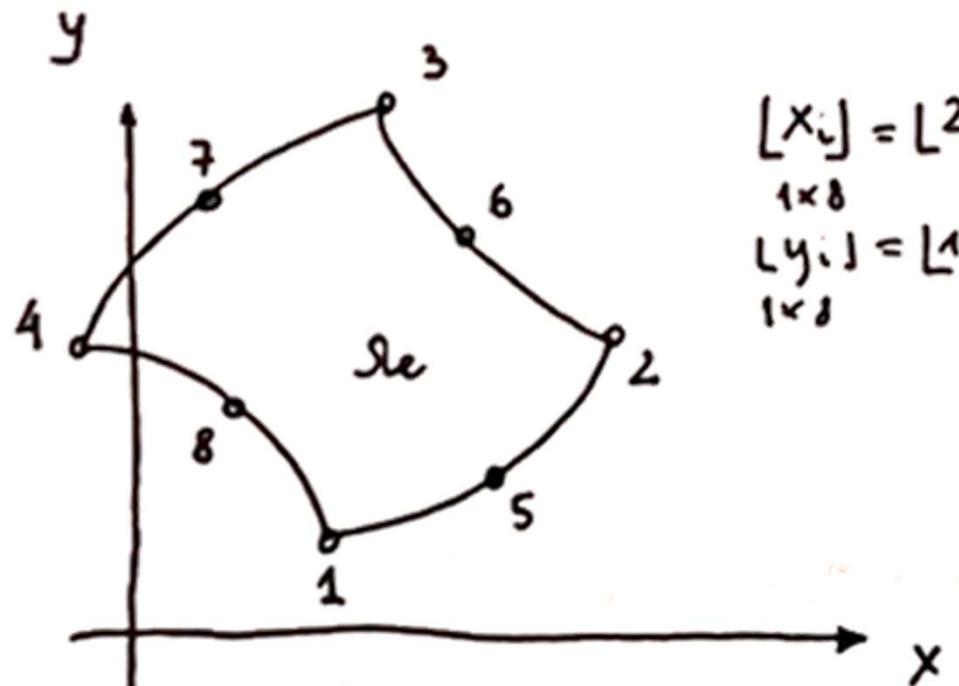


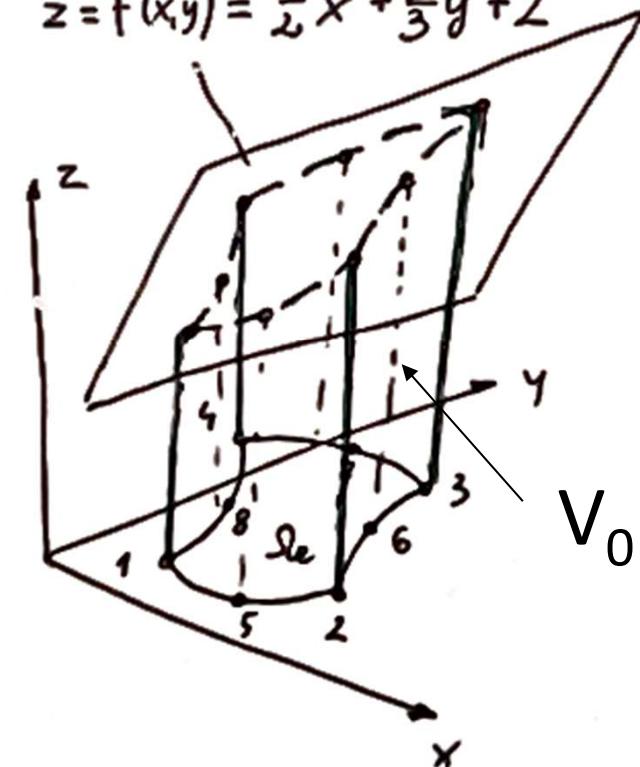
Example. QUAD-8node, numerical integration ($n = 3$). Find area of the finite element and volume V_0 .



$$[x_i] = [2, 7, 4, -1, 5, 5, 1, 1]^T$$

$$[y_i] = [1, 4, 9, 4, 2, 6, 7, 3]^T$$

$$z = f(x, y) = \frac{1}{2}x + \frac{2}{3}y + 2$$



Volume:

$$V_0 = \iint_A f(x,y) dx dy =$$

$$= \int_{-1}^1 \int_{-1}^1 \left(\frac{1}{2}x + \frac{2}{3}y + 2 \right) \det[J(\xi, \eta)] d\xi d\eta =$$

$$= \int_{-1}^1 \int_{-1}^1 \left(\left(\frac{1}{2} \ln \left| \begin{matrix} x_i \\ y_i \end{matrix} \right| + \frac{2}{3} \ln \left| \begin{matrix} y_i \\ x_i \end{matrix} \right| + 2 \right) \cdot \det[J(\xi, \eta)] \right) d\xi d\eta =$$

$$= \left| \det[J] = \frac{\partial x}{\partial \xi} \cdot \frac{\partial y}{\partial \eta} - \frac{\partial y}{\partial \xi} \cdot \frac{\partial x}{\partial \eta} = \frac{\partial \ln}{\partial \xi} \cdot \frac{\partial \ln}{\partial \eta} \left| \begin{matrix} x_i \\ y_i \end{matrix} \right| - \frac{\partial \ln}{\partial \xi} \cdot \frac{\partial \ln}{\partial \eta} \left| \begin{matrix} y_i \\ x_i \end{matrix} \right| \right| =$$

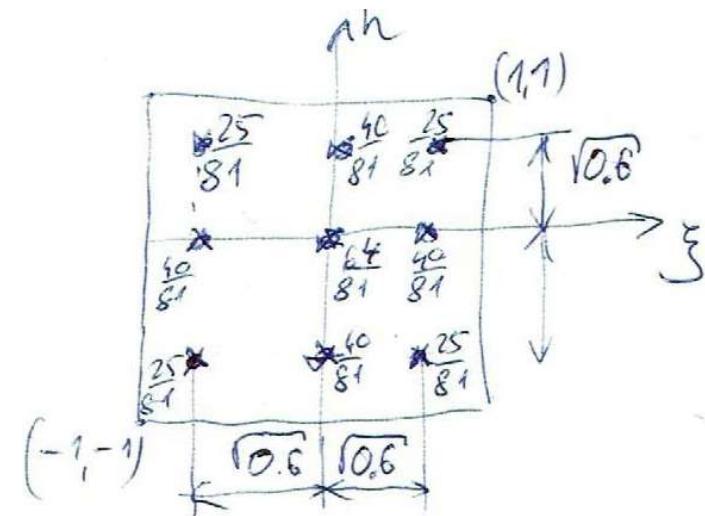
$$= \left(\frac{1}{2} \ln(-\sqrt{0.6}, \sqrt{0.6}) \cdot \left| \begin{matrix} x_i \\ y_i \end{matrix} \right| + \frac{2}{3} \ln(-\sqrt{0.6}, \sqrt{0.6}) \cdot \left| \begin{matrix} y_i \\ x_i \end{matrix} \right| + 2 \right) \cdot \det[J(-\sqrt{0.6}, \sqrt{0.6})] \cdot \frac{25}{81} +$$

$$+ \left(\frac{1}{2} \ln(0, -\sqrt{0.6}) \cdot \left| \begin{matrix} x_i \\ y_i \end{matrix} \right| + \frac{2}{3} \ln(0, -\sqrt{0.6}) \cdot \left| \begin{matrix} y_i \\ x_i \end{matrix} \right| + 2 \right) \cdot \det[J(0, -\sqrt{0.6})] \cdot \frac{40}{81} +$$

$$\dots + (\text{7 components}) = 220.4 \text{ mm}^3$$

$$x = \left[\begin{matrix} N \\ I \cdot S \end{matrix} \right] \cdot \left\{ \begin{matrix} x_i \\ y_i \end{matrix} \right\}$$

$$y = \left[\begin{matrix} N \\ I \cdot S \end{matrix} \right] \cdot \left\{ \begin{matrix} y_i \\ x_i \end{matrix} \right\}$$



Area: $A = \iint_{\text{Solid}} dy = \iint_{-1}^1 \det[J(\xi, \eta)] d\xi d\eta =$

$$= \det[J(-1, 1)] \cdot \frac{25}{81} + \det[J(0, 1)] \cdot \frac{40}{81} + \dots + (\text{7 components}) = 33.333 \text{ mm}^2$$